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N91-22322
J12

**An Integrated Control/Structure Design
Method using Multi-objective Optimization**

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Presented at the
4th NASA Workshop on Computational Control
of Flexible Aerospace Systems.

Kingsmill Resort, Williamsburg, Virginia.
July 11-13, 1990.

Structural design variables are the diameters of tubular beam elements. Hence, there are 10 structural design variables. Controller design variables determine the attitude and attitude rate gain matrices for the constant gain dissipative feedback control law. Since G_p and G_r have to be positive definite for this controller, we use Cholesky factors of the gain matrices as follows: $G_p = L_p L_p^T$, and $G_r = L_r L_r^T$, where L_p and L_r are lower triangular matrices. The controller design variables are the elements of L_p and L_r matrices. Subsequently, there are 24 controller design variables in this study.

Design Variables

Structural:

10 structural design variables -- diameters of tubular beam elements

Controller:

Controller design variables -- the attitude and rate gain matrices
 Since G_p , G_r , are positive definite, we use Cholesky factors for design

$$G_p = L_p L_p^T \quad \text{and} \quad G_r = L_r L_r^T$$

Hence, controller design variables are elements of L_p and L_r

24 controller variables for this case.

Four control/structure objectives were selected for this study. The first structural objective is to minimize mass of the flexible mast and the actuators, M_T ; obtained by subtracting the fixed masses of the shuttle and the reflector from the total mass of the system. Another structural objective is to maximize the first open-loop frequency, ω_1 , so that the structure can be made as stiff as possible within allowable values of the mass and the control objectives. The first control objective is to minimize a measure of transient response decay time, τ . This measure is the sum of the reciprocal of absolute real parts of the closed-loop eigenvalues, as shown below. The last CSI objective is to minimize a noise attenuation measure, σ , which is the steady-state root mean square attitude error due to a white noise input at the sensors.

CSI Performance Objectives

Structural:

- 1) Mass of flexible mast and actuator mass, M_T
- 2) First open-loop structural frequency, ω_1

Controller:

- 3) A measure of transient response decay time, τ

$$\tau = \sum_{i=1}^n \frac{1}{|\operatorname{Re} \lambda_i|}$$

- 4) A noise attenuation measure -- root mean square attitude error due to a white noise input at the sensors, σ

The main issue in using approaches which employ incremental scaling parameters a_j and b_j is to select values for these parameters. Although arbitrary values of these parameters would still lead to a Pareto optimal solution with our approach, some preliminary optimizations are performed to establish trends in the behavior of various objectives, which assist in choosing reasonable values for the scaling parameters. Minimizing mass, M_T , alone takes the structural variables to their lower bounds (for least possible structural mass), and the controller variables close to zero (for least actuator mass). However, this makes the structure very flexible. Maximizing first open-loop frequency, ω_1 , sends the structural variables to their upper bounds and the controller variables close to zero (near zero point masses at the tips). This leads to a very massive and stiff structure. These optimizations show the tradeoff between the first two objectives. Optimizing both, i.e. minimizing mass and maximizing the first open-loop frequency, simultaneously, results in reasonable values for both objectives.

Trends from Optimization

- * Minimizing mass M_T :
 structural variables --> lower bounds
 controller variables --> close to zero
 Mass $M_T \sim 10^{-4}$
 very flexible structure, $\omega_1 \sim 10^{-3}$; $\tau \sim 10^9$; $\sigma = 1.65 \times 10^{-5}$
- * Maximizing first open loop frequency ω_1 :
 structural variables --> upper bounds
 controller variables --> close to zero
 First open loop frequency $\omega_1 = 12.24$
 very massive structure, $M_T \sim 15300$, $\tau = 134.26$, $\sigma = 2.3 \times 10^{-5}$
- * Structural optimization -- minimizing mass, maximizing frequency :
 structural variables -- thick close to shuttle end, thin out at the reflector end
 controller variables -- close to zero
 $M_T = 2827.3$, $\omega_1 = 0.173$,
 $\tau = 99.9$, $\sigma = 2.15 \times 10^{-5}$

The previous optimizations were primarily structural optimizations. In controls, first a rigid body controller is designed (with no optimization) as follows: $G_p = \omega^2 J$ and $G_r = 2\rho\omega J$, where J is the moments of inertia matrix for the structure and ρ, ω are closed-loop damping ratio and frequency. With $\rho = 0.707$ and $\omega = 0.05$ for the nominal SCOLE configuration, we get $\tau = 121.34$ and $\sigma = 2.16 \times 10^{-5}$. Next, an optimization is performed with respect to control variables only, while using the nominal structural configuration for SCOLE, resulting in reduction of τ to 104.37. Finally, performing simultaneous optimization with respect to structural and controller variables reduces τ further to 65.86, but this results in a massive and stiff structure with $M_T = 5537.6$, since there was no restriction on mass. Thus, there is another tradeoff involved between mass, M_T , and the transient response decay measure, τ .

Minimizing transient response decay time measure, τ :

- 1) No optimization. Using rigid body controller as
 $G_p = \omega^2 J$ and $G_r = 2\rho\omega J$
 where $\omega = 0.05$ and $\rho = 0.707$
 $\tau = 121.34$ and $\sigma = 2.16 \times 10^{-5}$
- 2) Optimization with controller variables only.
 structural variables at nominal value
 $\tau = 104.37$ and $\sigma = 2.12 \times 10^{-5}$
- 3) Optimization with both structural and controller variables
 $\tau = 65.86$ and $\sigma = 2.16 \times 10^{-5}$
 but since there is no restriction on mass, $M_T = 5537.67$

With some idea of the tradeoffs among different objectives and some insight into the numerical values of the objectives involved for selecting the parameters a_j and b_j , multi-objective optimizations are performed, next. The control-optimized design is used as the initial design. In order to reduce both mass and response decay time, lower values of desired mass and desired response decay time are used. The desired values for the first open-loop frequency and the noise attenuation measure, are used more as constraint values than performance objectives. The parameters b_j were chosen to make the incremental variations from the desired values commensurable. The optimization results in lower values for both mass, $M_T = 2847.7$, and response decay time measure, $\tau = 94.622$. However, the first open-loop frequency is lower than its desired value. To emphasize this objective more, we reduce the value of parameter b_2 . Now, the optimal first open-loop frequency is much closer to its desired value; but the mass and the response decay time measure, are not reduced quite as much.

Multi-objective Optimization Results

	M_T	ω_1	τ	σ
INITIAL	2881.0	0.2055	104.37	2.1×10^{-5}
a_1^1	2800.0	0.2	90.0	2.2×10^{-5}
b_1^1	10.0	0.01	1.0	10^{-4}
f_{opt}^1	2847.7	0.153	94.622	2.14×10^{-5}
a_1^2	2800.0	0.2	90.0	2.2×10^{-5}
b_1^2	10.0	0.001	1.0	10^{-4}
f_{opt}^2	2880.6	0.197	97.92	2.21×10^{-5}

Previous results demonstrated that mass was the constraining factor in improving the response decay time. In fact, the optimizer was reducing the structural mass making the mast more flexible and adding the mass to the actuators to improve τ . Therefore, in the next series of optimizations, the desired mass is increased to allow reduction in response decay time. Optimization results indeed show the trend of the mass going up while τ is reduced. In a similar manner, by varying the values of a_j and b_j , the designer can place different emphasis on various objectives, and perform parametric tradeoff studies with Pareto optimal designs.

Multi-objective Optimization Results

	M_T	ω_1	τ	σ
INITIAL	2881.0	0.2055	104.37	2.1×10^{-5}
a_j^3	3000.0	0.2	90.0	2.2×10^{-5}
b_j^3	10.0	0.01	1.0	10^{-4}
f_{opt}^3	2996.11	0.292	89.74	2.15×10^{-5}
a_j^4	3500.0	0.2	80.0	2.2×10^{-5}
b_j^4	10.0	0.01	1.0	10^{-4}
f_{opt}^4	3575.2	0.485	87.53	2.15×10^{-5}

This paper demonstrates the benefits of a multi-objective optimization-based control/structure integrated design methodology. An application of the proposed CSI methodology to the integrated design of the SCOLE configuration is presented here. Integrated design resulted in reducing both the control performance measure, τ , and the mass, M_T . Thus, better overall performance is achieved through integrated design optimization.

The multi-objective optimization approach used here provides Pareto optimal solutions by unconstrained minimization of a differentiable KS function. Furthermore, adjusting the parameters a_j and b_j gives insight into the trade-offs involved between different objectives.

Concluding Remarks

- * Control/Structure Integrated Design:
 - > Example application of CSI methodology to SCOLE
 - > Integrated design optimization gives better overall performance
- * Multi-objective Optimization Approach :
 - > Pareto optimal solutions
 - > Unconstrained optimizations
(constraints can be included as desired values with large weights)
 - > Adjusting a_j and b_j gives insight into tradeoffs involved between different objectives

Benefits of the CSI design were observed in a structurally simple SCOLE configuration. Greater opportunity for such benefits exists in the CSI design of more complex space structures. We will be applying this methodology to: 1) the EOS (Earth Observing System) structure, which is the ADMT/CSI focus configuration, and 2) the phase-zero evolutionary model at NASA Langley Research Center. This methodology will also be used with more sophisticated control laws such as dynamic dissipative controllers, as well as, LQG and H_∞ optimal controllers. Also, open-loop plant dynamics could be refined by including sensor/actuator dynamics (which would include filtering of input and output signals).

Future Work

- * Apply this technique to more complex structures
 - 1) EOS structure --> ADMT/CSI focus configuration
 - 2) Phase zero evolutionary model
- * Use with more sophisticated control laws
 - 1) Dynamic dissipative controllers
 - 2) LQG and H_∞ optimal controllers
- * Optimization including sensor/actuator dynamics
(which would include filtering of input and output signals)

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